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Resonance in Series L-C-R :-

If in any ~~R~~ L-C-R circuit

$$V_L = V_C$$

$$I X_L = I X_C$$

$$X_L = X_C$$

$$X_L - X_C = 0$$

Reactance of circuit $z = 0$

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\boxed{Z = R}$$

i.e impedance of the circuit will become ~~max~~ minimum

$$I = \frac{V}{Z} = \frac{V}{R} \text{ (Maximum)}$$

It is the State of Resonance.

Resonant frequency :-

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi\nu = \frac{1}{\sqrt{LC}}$$

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

Sharpness of Resonance:-

We know

$$I_0 = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$$

In state of resonance $\therefore$

$$I_0^{max} = \frac{V_0}{R}$$

$\omega = \omega_0$ (Resonant frequency

Now, we are choosing two side frequencies $\omega_1 = \omega_0 + \Delta\omega$ and $\omega_2 = \omega_0 - \Delta\omega$ Such that the

$$I_0 = \frac{1}{\sqrt{2}} I_0^{max}$$

$$I_0 = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$$

$$= \frac{1}{\sqrt{2}} I_0^{max} = \frac{V_0}{\sqrt{2} R}$$

$$\Rightarrow \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{2} R$$

$$(X_L - X_C)^2 + R^2 = 2R^2$$

$$(X_L - X_C)^2 = R^2$$

$$X_L - X_C = R$$

$$\omega_1 L - \frac{1}{\omega_1 C} = R$$

$$(\omega_0 + \Delta\omega)L - \frac{1}{(\omega_0 + \Delta\omega)C} = R$$

$$\omega_0 L\left(1 + \frac{\Delta\omega}{\omega}\right) - \frac{1}{\omega_0 C\left(1 + \frac{\Delta\omega}{\omega}\right)} = R$$

$$\omega_0 L\left(1 + \frac{\Delta\omega}{\omega}\right) - \frac{\omega_0 L}{\left(1 + \frac{\Delta\omega}{\omega}\right)} = R$$

$$\omega_0 L\left[\left(1 + \frac{\Delta\omega}{\omega_0}\right) - \left(1 + \frac{\Delta\omega}{\omega_0}\right)^{-1}\right] = R$$

$$\frac{\Delta\omega}{\omega} <<< 1$$

$$(1+x)^n = 1 + nx$$

$$\omega_0 L\left[\left(1 + \frac{\Delta\omega}{\omega_0}\right) - \left(1 - \frac{\Delta\omega}{\omega_0}\right)\right] = R$$

$$\omega_0 L \frac{2\Delta\omega}{\omega_0} = R$$

$$L\,2\Delta\omega = R$$

$$2\Delta\omega = \frac{R}{L}$$

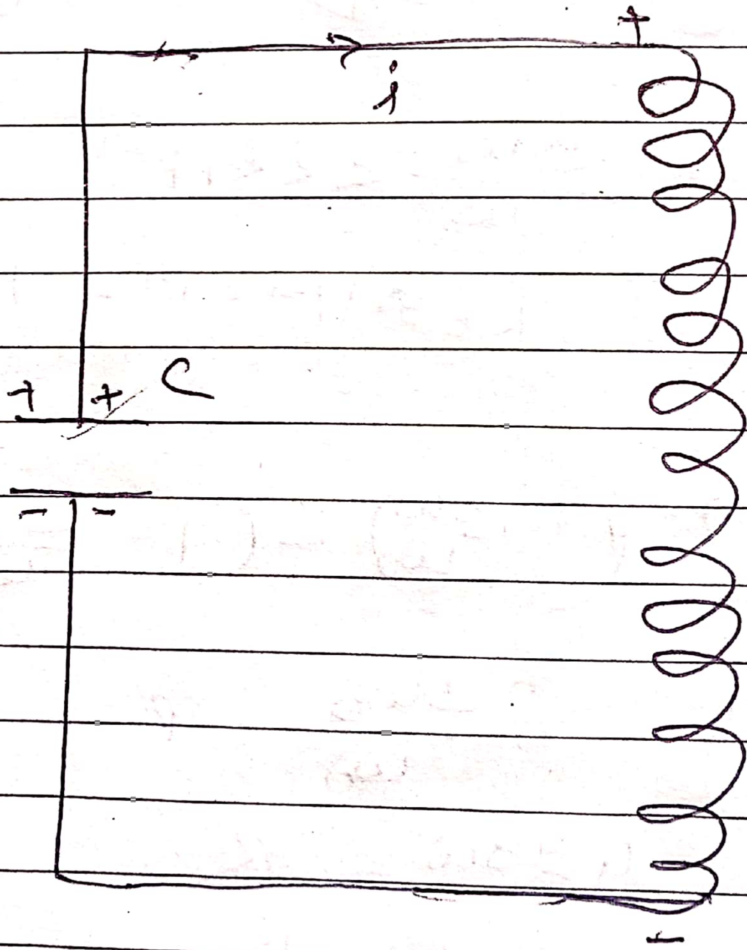
Sharpness of Resonance

$$Q = \frac{\omega_0}{2\Delta\omega}$$

$$\boxed{Q = \frac{\omega_0 L}{R}} = \frac{X_L}{R}$$

or $$\boxed{Q = \frac{1}{\omega_0 C R}} = \frac{X_C}{R}$$

L-C oscillation



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A Capacitor of capacity 'C' when fully charged is connected to inductor 'L' when charge defuses then back e.m.f is developed

$$V_C + V_L = 0$$

$$\frac{q}{C} - L\frac{dI}{dt} = 0$$

$$\frac{q}{LC} - \frac{dI}{dt} = 0$$

$$I = -\frac{dq}{dt} \qquad \frac{dI}{dt} = -\frac{d^2q}{dt^2}$$

$$\frac{q}{LC} + \frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

$$\frac{d^2q}{dt^2} = -\omega^2 q$$

On comparision to

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

for S.H.M

flow of charge will execute S.H.M.

$$\boxed{q = q_0 \cos\omega t}$$

$$I = -\frac{dq}{dt}$$

$$= -q_0 \frac{d\cos\omega t}{dt}$$

$$I = +q_0\omega \sin\omega t$$

$$\sin\omega t = 1$$

$$I_0 = q_0\omega$$

$$\boxed{I = I_0 \sin\omega t}$$

Stored electric energy at any instant

$$U_E = \frac{q^2}{2C} \quad \left(U_{E(max)} = \frac{q_0^2}{2C}\right)$$

Stored magnetic energy at any instant

$$U_m = \frac{1}{2} L I^2$$

$$\boxed{U_{m(max)} = \frac{1}{2} L I_0^2}$$

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