① $x_1 = ₹.21$ $x_2 = ₹.23$ $x_3 = ₹.20$
$x_4 = ₹.22$ $x_5 = ₹.25$

$$x_{mean} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{₹.21 + ₹.23 + ₹.20 + ₹.22 + ₹.25}{5}$$

$$= \frac{3.6.11}{5}$$

$$= ₹.22$$

$\Delta x_1 = |x_m - x_1| = 0.01$
$\Delta x_2 = |x_m - x_2| = 0.01$
$\Delta x_3 = |x_m - x_3| = 0.02$
$\Delta x_4 = |x_m - x_4| = 0.00$
$\Delta x_5 = |x_m - x_5| = 0.03$

$$\Delta x_{mean} = \frac{\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4 + \Delta x_5}{5}$$

$$= \frac{0.01 + 0.01 + 0.02 + 0.00 + 0.03}{5}$$

$$= \frac{0.07}{5}$$

$$= 0.01$$

true measure value

$x = x_{mean} \pm \Delta x_{mean}$

$x = 7.22 \pm 0.01$

Relative error $= \frac{\Delta x_{mean}}{x_{mean}}$

$= \frac{0.01}{7.22}$

% error = Relative error $\times 100$

$= \frac{\Delta x_{mean}}{x_{mean}} \times 100$

$= \frac{0.01}{7.22} \times 100$

$=$

Combination of errors:

(a) error of sum or difference

Let $Z = A + B$

A & B are individual measurements and Z is the sum.

If ΔA & ΔB are the individual errors in A & B respectively then if let ΔZ is the error in sum.

$$Z \pm \Delta Z = A \pm \Delta A + B \pm \Delta B$$

$$Z \pm \Delta Z = A + B \pm \Delta A \pm \Delta B$$

$$\underset{\wedge}{Z} \pm \Delta Z = \underset{\wedge}{Z} \pm \Delta A \pm \Delta B$$

$$\pm \Delta Z = \pm \Delta A \pm \Delta B$$

for maximum error

$$\pm \Delta Z = \pm (\Delta A + \Delta B)$$

i.e $\boxed{\Delta Z = \Delta A + \Delta B}$

So maximum error in the sum and difference, the sum of individual errors is the maximum error.

Q. Perimeter of Rectangle

$l = 3.21 \pm 0.05$ cm

$b = 2.51 \pm 0.04$ cm

$P = 2(l + b)$

$\Delta P = 2(\Delta l + \Delta b)$

$= 2(0.05 + 0.04)$

$= 2(0.09)$

$= (0.18)$

$P = 2(l + b) \pm \Delta P$

$= 2(5.72) \pm 0.18$

$P = 11.44 \pm 0.18$

(b) error of product or quotient:-

Let $Z = AB$

If ΔA & ΔB are individual errors

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$

Where ΔZ is maximum error in product.

$$Z \pm \Delta Z = AB \pm B\Delta A \pm A\Delta B \pm \Delta A \Delta B$$

ΔA & ΔB being very small, there product $\Delta A \Delta B$ can be ignored.

$$Z \pm \Delta Z = AB \pm B\Delta A \pm A\Delta B$$

$$Z \pm \Delta Z = Z \pm B\Delta A \pm A\Delta B$$

$$\pm \Delta Z = \pm B\Delta A \pm A\Delta B$$

divide, $Z = AB$

$$\frac{\pm \Delta Z}{Z} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

for maximum error

$$\frac{\pm \Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

So, the maximum Relative error in product is sum of Relative errors in the individual measurement.

Note:-

$$Z = ABC$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$$

(c) error in power

$$Z = A^2$$

$$Z = A \cdot A$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta A}{A}$$

$$\boxed{\frac{\Delta Z}{Z} = 2\frac{\Delta A}{A}}$$

$$Z = A^3$$

$$\boxed{\frac{\Delta Z}{Z} = 3\frac{\Delta A}{A}}$$

In General

$$z = A^n$$

$$\boxed{\frac{\Delta z}{z} = n \frac{\Delta A}{A}}$$

Note:-

$$z = \frac{A^n B^p}{C^q}$$

$$\boxed{\frac{\Delta z}{z} = n \frac{\Delta A}{A} + p \frac{\Delta B}{B} + q \frac{\Delta C}{C}}$$

Q $V = (100 \pm 5)$ volt

$I = (10 \pm 0.2)$ amp

$R = \frac{V}{I} = \frac{100}{10} = 10\,\Omega$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$\frac{\Delta V}{V} = \frac{5}{100} = \frac{1}{20}$

$\frac{\Delta I}{I} = \frac{0.2}{10} = \frac{1}{50}$

$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

$\frac{\Delta R}{10} = \frac{1}{20} + \frac{1}{50}$

$\Delta R = \frac{1}{2} + \frac{1}{5}$

$\Delta R = 0.5 + 0.2$

$\Delta R = 0.7$

Relative error: $\frac{\Delta R}{R} = \frac{0.3}{10} = \frac{3}{100}$

% error $= \frac{3}{100} \times 100 = 3\%$

(a) % error in $m = 1\%$

% error in $v = 2\%$

% error in K.E $= 4$

$K = \frac{1}{2} m v^2$

$\frac{\Delta K}{K} = \frac{\Delta m}{m} + 2\frac{\Delta v}{v}$

$\frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta v}{v} \times 100$

% in K = % in m + 2 × % in v

$= 1\% + 2 \times 2\%$

$= 1\% + 4\%$

$= 5\%$

Q:- In, the volume of a sphere radius increases by 2%, what is the % increasement in its volume.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$$

$$\frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100$$

$$\% \text{ in } V = 3 \times \% \text{ in } r$$

$$= 3 \times 2\%$$

$$= 6\%$$

(Q.1) if in the measurement of length % error is 2% and in the breadth it is 3% what is the % error in Area.

$$A = lb$$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$