Exercise (1.1)

Question 1:

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Solution 1:

Consider the definition of a rational number.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and q $\neq$ 0.

- Zero can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5}$….
- Zero can be written as well $\frac{0}{-1}, \frac{0}{-2}, \frac{0}{-3}, \frac{0}{-4}$ ….

So, we arrive at the conclusion that 0 can be written in the form of $\frac{p}{q}$, where p and q are integers (q can be positive or negative integers).

Therefore, zero is a rational number.

Question 2:

Find six rational numbers between 3 and 4.

Solution 2:

We know that there are infinite rational numbers between any two numbers. As we have to find 6 rational numbers between 3 and 4

So multiply and divide by 7 (or any number greater than 6)

We get, $3 = 3\times \frac{7}{7} = \frac{21}{7}$

$$4 = 4 \times \frac{7}{7} = \frac{28}{7}$$

Thus the 6 rational numbers are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

Question 3:

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution 3:

We know that there are infinite rational numbers between any two numbers.

As we have to find 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

So, multiply and divide by 6 (Or any number greater than 5)

$$\frac{3}{5} = \frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$$

Thus the 5 rational numbers are $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

Question 4:

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Solution 4:

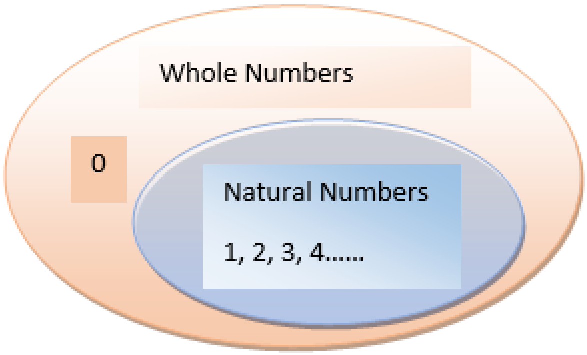
(i) Consider the whole numbers and natural numbers separately.

We know that whole number series is 0,1, 2, 3, 4,5..... .

We know that natural number series is 1, 2, 3, 4,5..... .

So, we can conclude that every natural number lie in the whole number series.

Diagrammatically, we can represent as follows:



Therefore, we conclude that, yes every natural number is a whole number.

(ii) Consider the integers and whole numbers separately.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$, where q = 1.

Now, considering the series of integers, we have – 4, – 3, – 2, – 1, 0, 1, 2,3, 4..... .

We know that whole number are 0, 1, 2, 3, 4, 5..... .

We can conclude that whole number series lie in the series of integers. But every integer does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

But, clearly every whole number is an integer.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$,where q $\neq$ 0

We know that whole numbers are 0, 1, 2, 3, 4, 5..... .

We know that every whole number can be written in the form of $\frac{p}{q}$ as follows

$\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}....$

We conclude that every whole number is a rational number.

But, every rational number (1/2, 1/3, 1/4,1/5,1/6….) is not a whole number. Therefore, we conclude that every rational number is not a whole number.

But, clearly every whole number is a rational number.

Exercise (1.2)

Question 1:

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Solution 1:

(i) Consider the irrational numbers and the real numbers separately.

- The irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0. (Eg: $\sqrt{2}, 3\pi$, .011011011…)

- The real number is the collection of rational numbers and irrational numbers.

Therefore, we conclude that, every irrational number is a real number.

(ii) Consider a number line. on a number line, we can represent negative as well as positive numbers.

- Positive numbers are represented in the form of $\sqrt{1}, \sqrt{1.1}, \sqrt{1.2}$ ... ...
- But we cannot get a negative number after taking square root of any number.

(Eg: $\sqrt{-5}$ = 5i is a complex number (which you will study in higher classes))

Therefore, we conclude that every number point on the number line is not of the form $\sqrt{m}$, where m is a natural number.

(iii) Consider the irrational numbers and the real numbers separately.

- Irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0.
- A real number is the collection of rational numbers (Eg: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ ……) and irrational numbers (Eg: $\sqrt{2}, 3\pi$, .011011011…).

So, we can conclude that every irrational number is a real number. But every real number is not an irrational number.

Therefore, every real number is not an irrational number.

Question 2:

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution 2:

- Square root of every positive integer will not yield an integer. (Eg: $\sqrt{2}, \sqrt{3}, \sqrt{6 \ldots.}$) which are called irrational numbers

But $\sqrt{4}$ is 2, which is an integer.

Therefore, we conclude that square root of every positive integer is not an irrational number.

Question 3:

Show how $\sqrt{5}$ can be represented on the number line.

Solution 3:

We need to draw a line segment AB of 2 unit on the number line. Then draw a perpendicular line segment BC at B of 1 units. Then join the points C and A, to form a line segment AC. According to Pythagoras Theorem

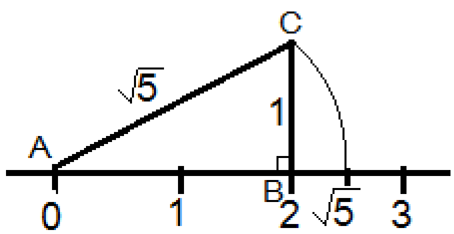
$AC^2 = AB^2 + BC^2$

$AC^2 = 2^2 + 1^2$

$AC^2 = 4 + 1 = 5$

$AC = \sqrt{5}$

Then draw the arc ACD, to get the number $\sqrt{5}$ on the number line.



Exercise (1.3)

Question 1:

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

(ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$

(v) $\frac{2}{11}$

(vi) $\frac{329}{400}$

Solution 1:

(i) $\frac{36}{100}$

On dividing 36 by 100, we get

$$\begin{array}{r} 0.36 \\ 100\overline{)\;36} \\ \underline{-0} \\ 360 \\ \underline{-300} \\ 600 \\ \underline{-600} \\ \underline{0} \end{array}$$

Therefore, $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii) $\frac{1}{11}$

On dividing 1 by 11, we get

$$
\begin{array}{r}
0.0909\ldots \\
11\overline{)\ 1} \\
\underline{-0} \\
10 \\
\underline{-0} \\
100 \\
\underline{-99} \\
10 \\
\underline{-0} \\
100 \\
\underline{-99} \\
1
\end{array}
$$

We observe that while dividing 1 by 11, the quotient = 0.09 is repeated.

Therefore, $\frac{1}{11}$ = 0.0909..... or $\frac{1}{11} = 0.\overline{09}$, which is a non-terminating and recurring decimal.

(iii) $4\frac{1}{8} = 4 + \frac{1}{8} = \frac{32+1}{8} = \frac{33}{8}$

On dividing 33 by 8, we get

$$
\begin{array}{r}
4.125 \\
8\overline{)\ 33} \\
\underline{-32} \\
10 \\
\underline{-8} \\
20 \\
\underline{-16} \\
40 \\
\underline{-40} \\
\underline{0}
\end{array}
$$

while dividing 33 by 8, the remainder is 0.

Therefore, $4\frac{1}{8} = \frac{33}{8}$ = 4.125, which is a terminating decimal.

(iv) $\frac{3}{13}$

On dividing 3 by 13, we get

$$
\begin{array}{r r}
 & 0.230769.... \\
13) & 3 \\
 & \underline{-0} \\
 & 30 \\
 & \underline{-26} \\
 & 40 \\
 & \underline{-39} \\
 & 10 \\
 & \underline{-0} \\
 & 100 \\
 & \underline{-91} \\
 & 90 \\
 & \underline{-78} \\
 & 120 \\
 & \underline{-117} \\
 & \underline{3}
\end{array}
$$

while dividing 3 by 13 the remainder is 3, which will continue to be 3 after carrying out 6 continuous divisions.

Therefore, $\frac{3}{13} = 0.230769.....$ or $\frac{3}{13} = 0.\overline{230769}$, which is a non-terminating and recurring decimal.

(v) $\frac{2}{11}$

On dividing 2 by 11, we get

$$
\begin{array}{r}
0.1818....\\
11\overline{)\quad\quad 2}\\
\underline{-0}\\
20\\
\underline{-11}\\
90\\
\underline{-88}\\
20\\
\underline{-11}\\
90\\
\underline{-88}\\
\underline{2}
\end{array}
$$

We can observe that while dividing 2 by 11, first the remainder is 2 then 9, which will continue to be 2 and 9 alternately.

Therefore, $\frac{2}{11} = 0.1818.....$ or $\frac{2}{11} = 0.\overline{18}$, which is a non-terminating and recurring decimal.

(vi) $\frac{329}{400}$

On dividing 329 by 400, we get

$$
\begin{array}{r}
0.8225 \\
400\overline{)\ 329} \\
\underline{-0} \\
3290 \\
\underline{-3200} \\
900 \\
\underline{-800} \\
1000 \\
\underline{-800} \\
2000 \\
\underline{-2000} \\
\underline{0}
\end{array}
$$

While dividing 329 by 400, the remainder is 0.

Therefore, $\frac{329}{400} = 0.8225$, which is a terminating decimal.

Question 2:

You know that $\frac{1}{7}$ = 0.142857..... . Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[*Hint*: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Solution 2:

$\frac{1}{7} = 0.\overline{142857}$ or $\frac{1}{7}$ = 0.142857.....

find the values of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$, without performing long division.

,$\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2\times\frac{1}{7}$, $3\times\frac{1}{7}$, $4\times\frac{1}{7}$, $5\times\frac{1}{7}$, and $6\times\frac{1}{7}$.

On substituting value of $\frac{1}{7}$ = 0.142857..... , we get

- $2\times\frac{1}{7}=2\times0.142857....=0.285714....$
- $3\times\frac{1}{7}=3\times0.142857....=0.428571....$
- $4\times\frac{1}{7}=4\times0.142857....=0.571428....$
- $5\times\frac{1}{7}=5\times0.142857....=0.714285....$
- $6\times\frac{1}{7}=6\times0.142857....=0.857142....$

Therefore, we conclude that, the values of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$, without performing long division, we get

- $\frac{2}{7}=0.\overline{285714}$
- $\frac{3}{7}=0.\overline{428571}$
- $\frac{4}{7}=0.\overline{571428}$
- $\frac{5}{7}=0.\overline{714285}$
- $\frac{6}{7}=0.\overline{857142}$.

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0.

(i) $0.\overline{6}$
(ii) $0.\overline{47}$
(iii) $0.\overline{001}$

Solution 3:

i. Let x = $0.\overline{6}\Rightarrow$ x = 0.6666..... …(1)

ii.

Multiply both sides by 10,

$10x =$ 0.6666 × 10

$10x = 6.6666......$ .....(2)

Subtracting (1) from (2), we get

$$\begin{array}{r} 10x = 6.6666..... \\ -x = 0.6666..... \\ \hline 9x = 6 \end{array}$$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

Therefore, on converting $0.\bar{6} = \frac{2}{3}$ which is in the $\frac{p}{q}$ form,

(i) Let $x = 0.4\overline{7} \Rightarrow x = 0.47777....$ ....(a)

Multiply both sides by 10, we get

$10x = 4.7777.....$ ....(b)

Subtract the equation (a) from (b),we get

$$\begin{array}{r} 10x = 4.7777..... \\ -x = 0.4777.... \\ \hline 9x = 4.3 \end{array}$$

$x = \frac{4.3}{9} \frac{\times 10}{\times 10} \quad \frac{43}{90}$

$x = \frac{43}{90}$.

Therefore, on converting $0.4\overline{7} = \frac{43}{90}$ in the $\frac{p}{q}$ form.

(iii) Let $x = 0.\overline{001} \Rightarrow$ ... (a)

multiply both sides by 1000 (because the number of recurring decimal number is 3)

$1000 \times x = 1000 \times 0.001001....$

So, $1000x = 1.001001.....$ ...(b)

Subtract the equation (a) from (b),

$$
\begin{array}{r}
1000x = 1.001001..... \\
-x = 0.001001.... \\
\hline
999x = 1
\end{array}
$$

$\rightarrow x = \frac{1}{999}$.

Therefore, on converting $0.\overline{001} = \frac{1}{999}$ which is in the $\frac{p}{q}$ form.

Question 4:

Express 0.99999.... in the form $\frac{p}{q}$. Are you surprised by your answer? Discuss why the answer makes sense with your teacher and classmates.

Solution 4:

Let x = 0.99999.... .....(a)

We need to multiply by 10 on both sides, we get

$10x$ = 9.9999.... .....(b)

Subtract the equation (a) from (b), to get

$$
\begin{array}{r}
10x = 9.99999..... \\
-x = 0.99999.... \\
\hline
9x = 9
\end{array}
$$

9x = 9 as $x = \frac{9}{9}$ or x = 1.

Therefore, on converting 0.99999...= $\frac{1}{1}$ which is in the $\frac{p}{q}$ form,

Yes, at a glance we are surprised at our answer.

But the answer makes sense when we observe that 0.9999……… goes on forever.

So, there is no gap between 1 and 0.9999……. and hence they are equal.

Question 5:

What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Solution 5:

We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

$$
\begin{array}{r|l}
17 & 1 \quad (0.0588235294117647....\\
 & \underline{-0}\\
 & 10\\
 & \underline{-0}\\
 & 100\\
 & \underline{-85}\\
 & 150\\
 & \underline{-136}\\
 & 140\\
 & \underline{-136}\\
 & 40\\
 & \underline{-34}\\
 & 60\\
 & \underline{-51}\\
 & 90\\
 & \underline{-85}\\
 & 50\\
 & \underline{-34}\\
 & 160\\
 & \underline{-153}\\
 & 70\\
 & \underline{-68}\\
 & 20\\
 & \underline{-17}\\
 & 30\\
 & \underline{-17}\\
 & 130\\
 & \underline{-119}\\
 & 110\\
 & \underline{-102}\\
 & 80\\
 & \underline{-68}\\
 & 120\\
 & \underline{-119}\\
 & \underline{1}
\end{array}
$$

We can observe that while dividing 1 by 17 we get 16 number of digits in the repeating block of decimal expansion which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that $\frac{1}{17}$ = 0.0588235294117647..... or $\frac{1}{17}$ = $0.\overline{0588235294117647}$,which is a non-terminating and recurring decimal.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ (q $\neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution 6:

Let us we take the examples $\frac{5}{2}, \frac{5}{4}, \frac{2}{5}, \frac{2}{10}, \frac{5}{16}$ of the form $\frac{p}{q}$ that are terminating decimals.

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{2}{10} = 0.2$

$\frac{5}{16} = 0.3125$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both. Therefore, q must satisfy in the form either 2^m or 5^n or both $2^m \times 5^n$ (where m= 0,1,2,3….. and n = 0,1,2,3….) in $\frac{p}{q}$ form .

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution 7:

All irrational numbers are non-terminating and non-recurring.

Eg: $\sqrt{2} = 1.41421 \ldots,$ $\sqrt{3} = 1.73205 \ldots \ldots$

$\sqrt{7}$= 2.645751……

Question 8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Solution 8:

Let us convert $\frac{5}{7}$ and $\frac{9}{11}$ into decimal form, we get

$\frac{5}{7}$ = 0.714285.... and $\frac{9}{11}$ = 0.818181....

Three irrational numbers that lie between0.714285.... and 0.818181.... are:

0.73073007300073….

0.74074007400074….

0.76076007600076….

Irrational numbers cannot be written in the form of p/q.

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

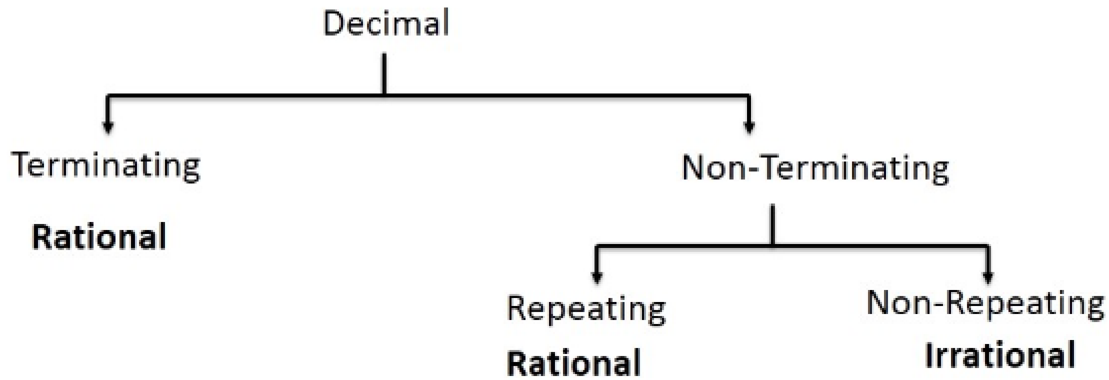
(iii) 0.3796

(iv) 7.478478...

(v) 1.101001000100001...

Solution 9:

We know



(i) $\sqrt{23}$

$\sqrt{23}$ = 4.795831…..
It is an irrational number

(ii) $\sqrt{225}=15$
Therefore $\sqrt{225}$ is a rational number.

(iii) 0.3796

It is terminating decimal. Therefore, it is rational number

(iv) 7.478478….

The given number 7.478478…. is a non-terminating recurring decimal, which can be converted into $\frac{p}{q}$ form.

While, converting 7.478478…. into $\frac{p}{q}$ form, we get

$x = 7.478478....$ … (a)
$1000x = 7478.478478....$ … (b)

While, subtracting (a) from (b), we get

$$\begin{aligned} 1000x &= 7478.478478.... \\ -x &= \quad\;\; 7.478478.... \\ \hline 999x &= 7471 \end{aligned}$$

$999x = 7471$

$$x = \frac{7471}{999}$$

Therefore, 7.478478…. is a rational number.

(v) 1.101001000100001....

We can observe that the number 1.101001000100001.... is a non-terminating non-recurring decimal. Thus, non-terminating and non-recurring decimals cannot be converted into $\frac{p}{q}$ form.

Therefore, we conclude that 1.101001000100001.... is an irrational number.

Exercise (1.4)

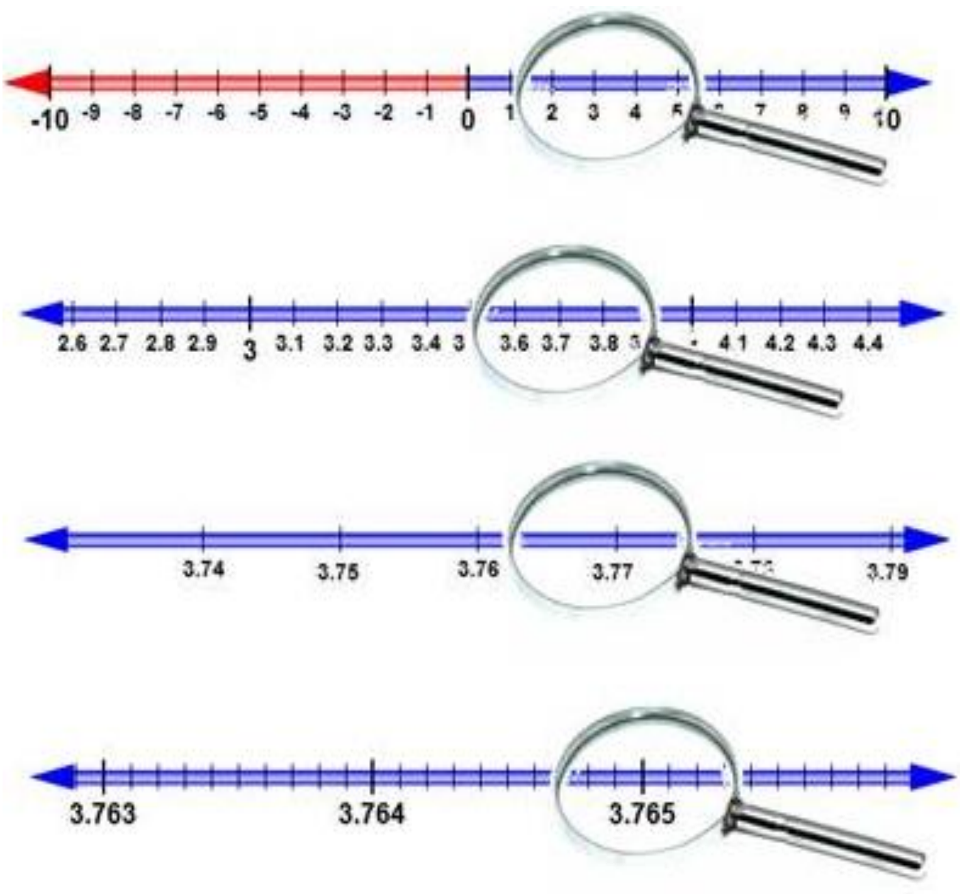
Question 1.

Visualize 3.765 on the number line using successive magnification.

Solution 1:

- Clearly, the value lies between 3 and 4
- The numbers 3.7 and 3.8 lie between 3 and 4.
- The numbers 3.76 and 3.77 lie between 3.7 and 3.8.
- The numbers 3.764 and 3.766 lie between 3.76 and 3.77
- The number 3.765 lies between 3.764 and 3.766.

Therefore, we need to use the successive magnification, after locating numbers 3 and 4 on the number line.



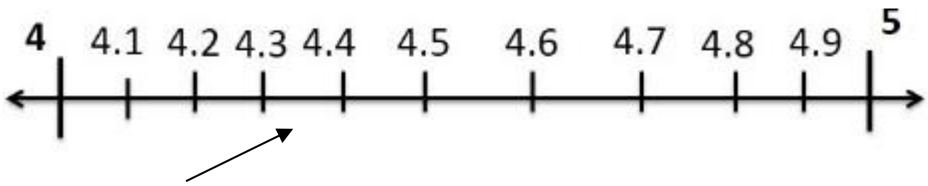
Question 2:

Visualize $4.\overline{26}$ on the number line, up to 4 decimal places.

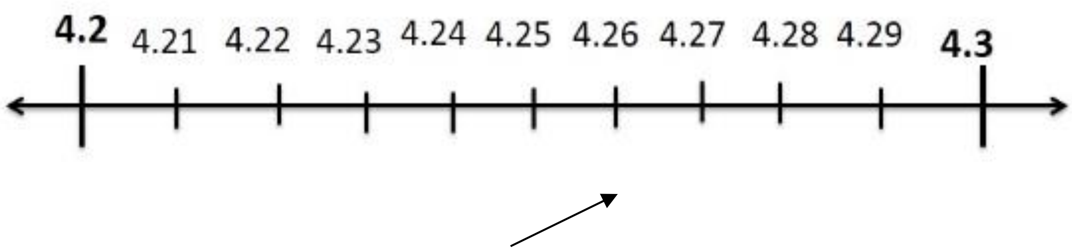
Solution 2:

The number $4.\overline{26}$ can also be written as 4.262.... .

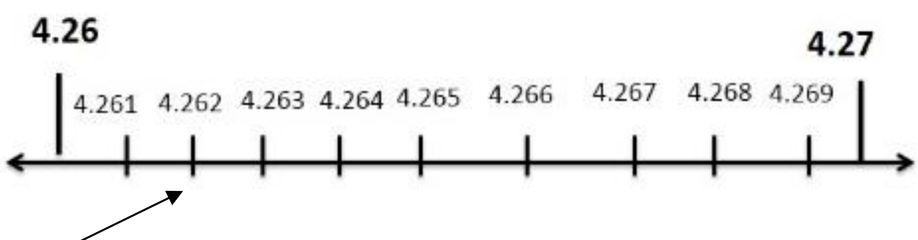
The number 4.2 lie between 4 and 5



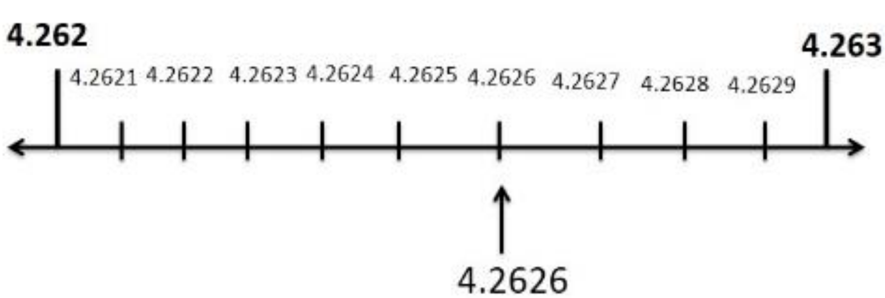
The numbers 4.26 lie between 4.2 and 4.3



The numbers 4.262 lie between 4.261 and 4.263



The number 4.2626 lie between 4.262 and 4.263



Therefore, we need to use the successive magnification, after locating numbers 4 and 5 on the number line to visualize up to 4 decimal places.

Exercise (1.5)

Question 1:

Classify the following numbers as rational or irrational:

i. $2-\sqrt{5}$

ii. π

iii. $\frac{2\sqrt{7}}{7\sqrt{7}}$

iv. $\frac{1}{\sqrt{2}}$

v. 2π

Solution 1:

(i) $2-\sqrt{5}$

$\sqrt{5}$ = 2.236...., is a non-terminating and non-repeating irrational number.

$2-\sqrt{5}$ = 2 – 2.236.... (By substituting the value of $\sqrt{5}$)

= –0.236...., which is also an irrational number.

Therefore, $2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23}$

$(3+\sqrt{23})-\sqrt{23}=3+\sqrt{23}-\sqrt{23}=3 = \frac{3}{1}$

which is in the form of $\frac{p}{q}$ and it is a rational number.

Therefore, we conclude that $(3+\sqrt{23})-\sqrt{23}$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$

Which is in the form of $\frac{p}{q}$ and it is a rational number.

Therefore, we conclude that $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

$\sqrt{2}$ =1.414...., is a non-terminating and non-repeating irrational number.

Therefore, we conclude that $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

$\pi = 3.1415....$, which is an irrational number.

Observe, Rational x Irrational = Irrational

Therefore 2π will also be an irrational number.

Question 2:

Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution 2 :

(i) $(3+\sqrt{3})(2+\sqrt{2})$

$(3+\sqrt{3})(2+\sqrt{2}) = 3(2+\sqrt{2})+\sqrt{3}(2+\sqrt{2})$ (multiplication on binomial by binomial)

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

Using the identity $(a+b)(a-b) = a^2 - b^2$

Therefore

$$(3+\sqrt{3})(3-\sqrt{3}) = 3^2-(\sqrt{3})^2$$
$$= 9-3 = 6$$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Using Identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2 + 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2$$
$$= 5+2\sqrt{10}+2$$
$$= 7+2\sqrt{10}$$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Using Identity (a – b)(a + b) = $a^2 - b^2$

$$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2$$
$$= 5 - 2 = 3$$

Question 3:

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \dfrac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution 3:

Here, pi = 22/7 is a rational no. but this is an approximate value.
If we divide 22 by 7, the quotient (3.14....) is a non-terminating non-recurring number. i.e. it is irrational.

Approximate fractions include (in order of increasing accuracy)
22/7, 333/106, 355/113, 52163/16604, 103993/33102, and 245850922/78256779

If we divide anyone of these we get, the quotient (3.14....) is a non-terminating non-recurring number. i.e. it is irrational

Therefore, either circumference (c) or diameter (d) or both can be irrational numbers.

Therefore, we can conclude that as such there is no contradiction regarding the value of π and we realize that the value of π is irrational.

Question 4. Represent $\sqrt{9.3}$ on the number line.

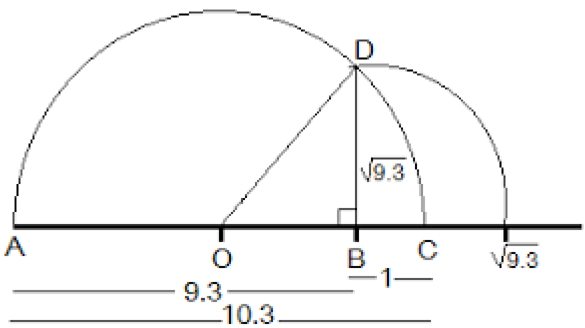
Solution 4:

- Mark the distance 9.3 units from a fixed-point A on a given line to obtain a point B

  such that AB = 9.3 units.
- From B mark a distance of 1 unit and mark the point as C.
- Find the mid-point of AC and mark the point as O.
- Draw a semi-circle with centre O and radius OC = 5.15 units

(AC = AB + BC = 9.3 + 1=10.3. Therefore OC = $\frac{AC}{2} = \frac{10.3}{2} = 5.15$).

- Draw a line perpendicular at B and draw an arc with centre B and let meet at semicircle AC at D
- BE = BD = $\sqrt{9.3}$ (Radius of an arc DE).



Question 5. Rationalize the denominators of the following:

i. $\frac{1}{\sqrt{7}}$.

ii. $\frac{1}{\sqrt{7}-\sqrt{6}}$

iii. $\frac{1}{\sqrt{5}+\sqrt{2}}$

iv. $\frac{1}{\sqrt{7}-2}$

Solution 5:

(i) $\frac{1}{\sqrt{7}}$

Multiply and divide by $\sqrt{7}$,

$$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Multiply and divide by $\sqrt{7}+\sqrt{6}$

We get,

$$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$$

Using $(a-b)(a+b) = a^2 - b^2$ in the denominator

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{1}$$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Multiply and divide by $\sqrt{5}-\sqrt{2}$ We get,

$$\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$

Using identity $(a-b)(a+b) = a^2 - b^2$ in the denominator

$$\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv) $\frac{1}{\sqrt{7}-2}$

Multiply and divide by $\sqrt{7}+2$. we get,

$$\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{(\sqrt{7}-2)(\sqrt{7}+2)}$$

Using identity $(a-b)(a+b) = a^2 - b^2$ in the denominator

$$\begin{aligned}\frac{1}{\sqrt{7}-2} &= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2} \\ &= \frac{\sqrt{7}+2}{7-4} \\ &= \frac{\sqrt{7}+2}{3}\end{aligned}$$

Exercise (1.6)

Question 1:

Find:

(i) $64^{\frac{1}{2}}$

(ii) $32^{\frac{1}{5}}$

(iii) $125^{\frac{1}{3}}$

Solution 1 :

(i) $64^{\frac{1}{2}}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$, where a > 0.

$64^{\frac{1}{2}} = \sqrt[2]{64} = \sqrt[2]{8\times 8} = 8$

Therefore, the value of $64^{\frac{1}{2}}$ is 8.

(ii) $32^{\frac{1}{5}}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$, where a > 0

$32^{\frac{1}{5}} = \sqrt[5]{32} = \sqrt[5]{2\times 2\times 2\times 2\times 2} = 2$

Alternatively :

Using $(a^m)^n = a^{mn}$

$$32^{\frac{1}{5}} = (2\times 2\times 2\times 2\times 2)^{\frac{1}{5}}$$
$$= (2^5)^{\frac{1}{5}}$$
$$= 2^{5\times\frac{1}{5}}$$
$$= 2$$

Therefore, the value of $32^{\frac{1}{5}}$ is 2.

(iii) $125^{\frac{1}{3}}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$, where a > 0.

$125^{\frac{1}{3}} = \sqrt[3]{125} = \sqrt[3]{5\times5\times5} = 5$

Therefore, the value of $125^{\frac{1}{3}}$ is 5.

Question 2:

Find:

(i) $9^{\frac{3}{2}}$

(ii) $32^{\frac{2}{5}}$

(iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Solution 2:

(i) $9^{\frac{3}{2}}$

We know that $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, where a > 0.

We conclude that $9^{\frac{3}{2}}$ can also be written as

$$\begin{aligned}\sqrt[2]{(9)^3} &= \sqrt[2]{9\times9\times9} \\ &= \sqrt[2]{3\times3\times3\times3\times3\times3} \\ &= 3\times3\times3 = 27\end{aligned}$$

Alternatively :

Using $(a^m)^n = a^{mn}$

$$9^{\frac{3}{2}} = (3\times3)^{\frac{3}{2}}$$
$$= (3^2)^{\frac{3}{2}}$$
$$= 3^{2\times\frac{3}{2}}$$
$$= 3^3 = 27$$

Therefore, the value of $9^{\frac{3}{2}}$ will be 27.

(ii) $32^{\frac{2}{5}}$

We know that $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, where a > 0.

We conclude that $32^{\frac{2}{5}}$ can also be written as

$$\sqrt[5]{(32)^2} = \sqrt[5]{(2\times2\times2\times2\times2)\times(2\times2\times2\times2\times2)}$$
$$= 2\times2$$
$$= 4$$

Therefore, the value of $32^{\frac{2}{5}}$ will be 4.

(iii) $16^{\frac{3}{4}}$

We know that $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, where a > 0.

We conclude that $16^{\frac{3}{4}}$ can also be written as

$$\sqrt[4]{(16)^3} = \sqrt[5]{(2\times2\times2\times2)\times(2\times2\times2\times2)\times(2\times2\times2\times2)}$$
$$= 2\times2\times2$$
$$= 8$$

Therefore, the value of $16^{\frac{3}{4}}$ will be 8.

Alternatively :

Using $(a^m)^n = a^{mn}$

$$16^{\frac{3}{4}} = (4\times4)^{\frac{3}{4}}$$
$$= (4^2)^{\frac{3}{4}}$$
$$= (4)^{2\times\frac{3}{4}}$$
$$= (2^2)^{2\times\frac{3}{4}}$$
$$= 2^{2\times2\times\frac{3}{4}} = 2^3 = 8$$

(iv) $125^{\frac{-1}{3}}$

We know that $a^{-n} = \dfrac{1}{a^n}$

We conclude that $125^{\frac{-1}{3}}$ can also be written as $\dfrac{1}{125^{\frac{1}{3}}}$ or $\left(\dfrac{1}{125}\right)^{\frac{1}{3}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where a > 0.

We conclude that $\left(\dfrac{1}{125}\right)^{\frac{1}{3}}$ can also be written as

$$\sqrt[3]{\left(\frac{1}{125}\right)} = \sqrt[3]{\left(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\right)} = \frac{1}{5}$$

Therefore, the value of $125^{\frac{-1}{3}}$ will be $\dfrac{1}{5}$.

Question 3:

Simplify:

(i) $2^{\frac{2}{3}}\cdot 2^{\frac{1}{5}}$

(ii) $\left(3^{\frac{1}{3}}\right)^7$

(iii) $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}}\cdot 8^{\frac{1}{2}}$

Solution 3:

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

We know that $a^m \times a^n = a^{m+n}$

We can conclude that $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{2}{3}+\frac{1}{5}}$

$2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

Therefore, the value of $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ will be $2^{\frac{13}{15}}$.

(ii) $\left(3^{\frac{1}{3}}\right)^7$

We know that $(a^m)^n = a^{mn}$

We conclude that $\left(3^{\frac{1}{3}}\right)^7$ can also be written as $\left(3^{\frac{7}{3}}\right)$.

(iii) $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

We know that $\dfrac{a^m}{a^n} = a^{m-n}$

We conclude that

$\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

Therefore, the value of $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ will be $11^{\frac{1}{4}}$.

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

We know that $a^m \times b^m = (a \times b)^m$.

We can conclude that $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$

$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (56)^{\frac{1}{2}}$

Therefore, the value of $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$ will be $(56)^{\frac{1}{2}}$.